Desirabilitiy functions in multicriteria optimization

Observations made while implementing `desiRe`

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11. August 2008

Multicriteria Optimization Problem

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Given:

Influence factors $X = (X_1, \ldots, X_n)'$

Objectives $Y_1, \ldots, Y_k$ with $Y_i = f_i(X) + \varepsilon_i \quad \varepsilon_i \sim N(0, \sigma_i)$

Goal:

$$\min_{X_1,\ldots,X_n} (Y_1, \ldots, Y_k)$$

Desirabilities

Idea

Map objective values to $[0, 1]$. Interpret 1 as *desirable* and 0 as *undesirable*

Types

One-sided Objective value should be as above or below a threshold

Two-sided Objective value should stay in between two thresholds

Applications

- Mostly in chemistry, chemical and mechanical engineering
- Optimization of production- or chemical processes
- Quality Control

Harrington Desirabilities

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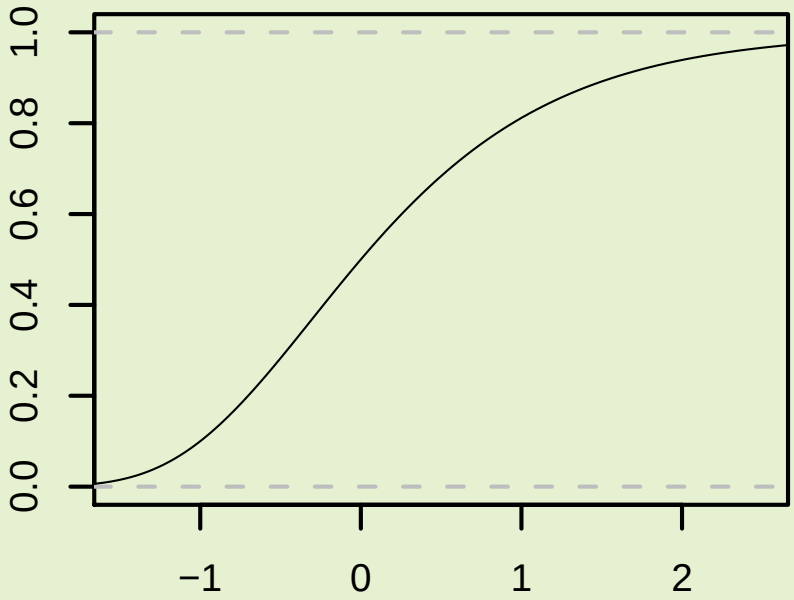
One-Sided

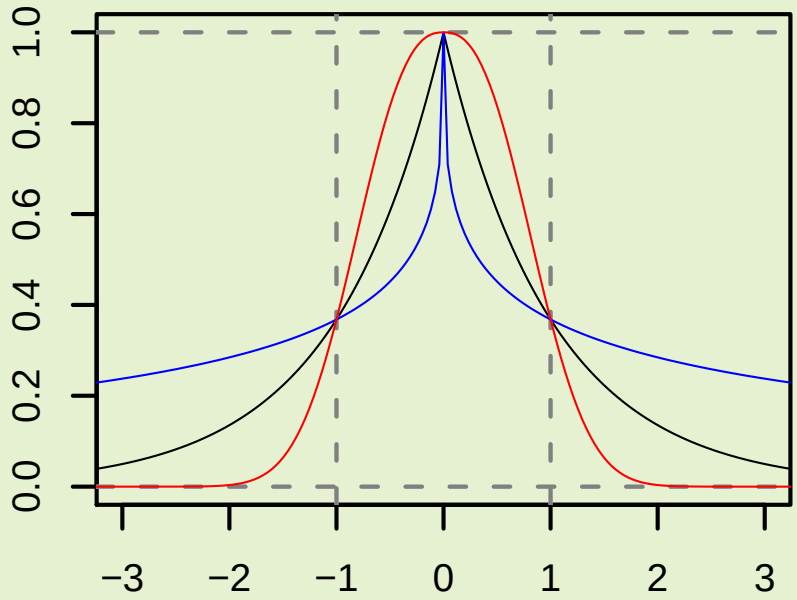


$$d(Y_i') = \exp(-\exp(-|Y_i'|^{n_i}))$$

$$Y_i' = b_0 + b_1 Y_i$$

Parameters: b_0, b_1

Two-Sided



$$d(Y_i') = \exp(-|Y_i'|^{n})$$

$$Y_i' = \frac{2Y_i - (USL + LSL)}{USL - LSL}$$

Parameters: LSL, USL, n

Derringer-Suich Desirabilities

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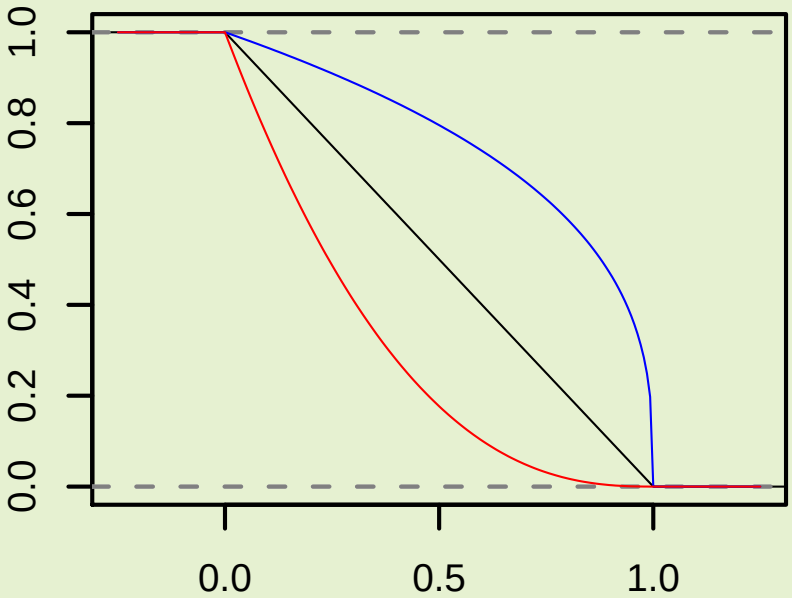
Parameters:

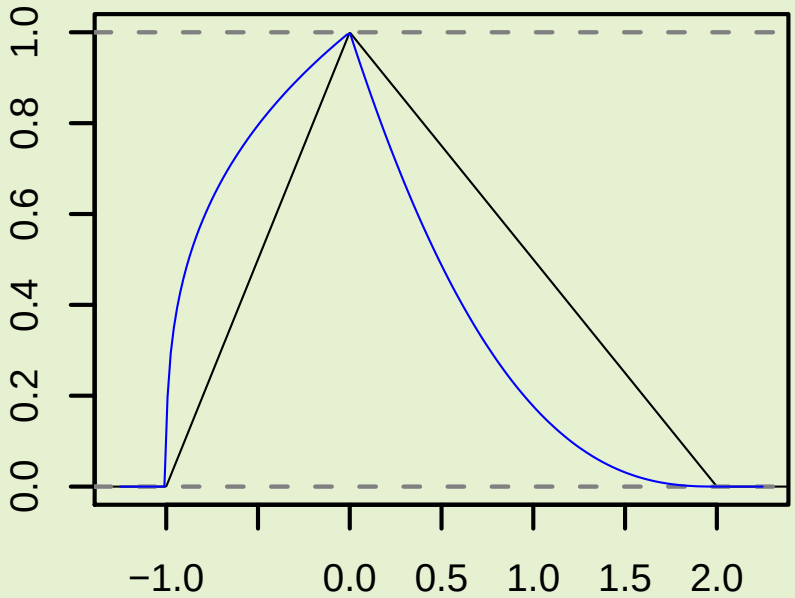
$$(LSL, T, USL, \beta_1, \beta_2)$$

One-Sided



Black: $(-\infty, 0, 1, 1, 1)$
Blue: $(-\infty, 0, 1, 1, \frac{1}{3})$
Red: $(-\infty, 0, 1, 1, 2.5)$

Two-Sided



Black: $(-1, 0, 2, 1, 1)$
Blue: $(-1, 0, 2, \frac{1}{3}, 2.5)$

Desirability Index (DI)

Geometric DI

$$D_g := \left(\prod_{i=1}^{k} d_i^{w_i}\right)^{1/\sum w_i}$$

Minimum DI

$$D_{\min} := \min_{1 \le i \le k} d_i$$

Mean DI

$$D_m := \frac{1}{\sum w_i} \sum_{i=1}^{k} w_i d_i$$

Solve MCO by maximizing appropriate DI.
Solve noisy MCO by maximizing expected value of appropriate DI.

Demonstration

Parameters

rt	Processing time	[min]
as	Shaft speed	[rpm]
E	Energy	[kWh]
$d90$	Particle size	$[\mu m]$
Fe	Iron content	$[\frac{mg}{kg}]$
η_{Ca}	Melting property	[Pa s]
τ_{Ca}	Melting property	[Pa]

C. Alamprese, L. Datei, Q. Semeraro (2007). Optimization of processing parameters of a ball mill refiner for chocolate. *Journal of Food Engineering*, 83(4), 629–636.

Things not shown:

- Explicit `d/p/q/r` functions for Harrington and Derringer-Suich desirabilities
- Control Chart functions (*in progress*)
- Plot functions
- Generalized Derringer-Suich desirabilities
- Linearization of Derringer-Suich desirabilities
- NSGA-II (see package `mco` on CRAN)
- Desirability Pareto Concept

Where to get it

`http://r-forge.r-project.org/projects/desire/`

or

CRAN (*coming soon*)